Super Stolarsky-3 Mean Labeling of Some Special Graphs

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Abstract

Let G = (V, E) be a graph with p vertices and q edges. Let **f**: V (G) → {1, 2, …, p + q} be an injective function. For a vertex labeling **f**, the induced edge labeling **f*** (e=uv) is defined by

$$f^*(e) = \left\lceil \sqrt{\frac{[f(u)^2+f(u)f(v)+f(v)^2]}{3}} \right\rceil \text{ (or) } \left\lfloor \sqrt{\frac{[f(u)^2+f(u)f(v)+f(v)^2]}{3}} \right\rfloor$$

. Then **f** is called a Super Stolarsky-3 Mean labeling if **f**(V(G))∪{**f**(e) / e ϵ E(G)} = {1, 2, …, p + q}. A graph which admits Super Stolarsky-3 Mean labeling is called Super Stolarsky-3 Mean graphs. In this paper we investigate Super Stolarsky-3 Mean Labeling of some special graphs.

Key words - Graph, Super Stolarsky-3 Mean labeling, Triangular ladder graph, Slanting ladder graph, Total graph, Middle graph, Balloon of Triangular snake graph and Balloon of Quadrilateral snake graph.

1. Introduction

All graphs G= (V, E) with p vertices and q edges are finite, simple and undirected. For all detailed survey of graph labeling, we refer to J.A.Gallian(2019) **[2]**. For all other standard terminology and notations we follow Harary**[3]**. The concept of Mean labeling has been introduced by R.Ponraj and S. Somasundaram in 2003**[5]**. S. Kavitha, S.S.Sandhya and E.Ebin Raja Merly introduced Stolarsky-3 Mean Labeling **[4]**.

In this paper we investigate Super Stolarsky-3 Mean labeling of Triangular ladder graph, Slanting ladder graph, Total graph, Middle graph, Balloon of Triangular snake graph and Balloon of Quadrilateral snake graph

The following definitions and theorems are necessary for our present investigation.

Definition 1.1: Let G = (V, E) be a graph with p vertices and q edges. Let **f**: V (G) → {1, 2, …, p + q} be an injective function. For a vertex labeling **f**, he induced edge labeling **f*** (e=uv) is defined by

$$f^*(e) = \left\lceil \sqrt{\frac{[f(u)^2+f(u)f(v)+f(v)^2]}{3}} \right\rceil \text{ (or) } \left\lfloor \sqrt{\frac{[f(u)^2+f(u)f(v)+f(v)^2]}{3}} \right\rfloor$$

. Then **f** is called a Super Stolarsky-3 Mean labeling if **f**(V(G))∪{**f**(e) / e ϵ E(G)} = {1, 2, …, p + q}. A graph which admits Super Stolarsky-3 Mean labeling is called Super Stolarsky-3 Mean graphs.

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Definition 1.2. A Triangular ladder is a graph obtained from L_n by adding the edges u_iv_{i+1}, $1 \leq i \leq n-1$, where u_i and v_i $1 \leq i \leq n$ are the vertices of L_n such that $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ are two paths of length n in the graph L_n

Definition 1.3. The Slanting ladder SL_n is a graph obtained from two paths $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ by joining each u_i with v_{i+1} ,$1 \leq i \leq n-1$.

Definition 1.4. The Total graph T(G) of graph G is the graph whose vertex set is V(G)∪E(G) and two vertices are adjacent whenever they are either adjacent or incident in G.

Definition 1.5. The Middle graph M(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

Definition 1.6. The Balloon of the Triangular snake $T_n(C_m)$ is the graph obtained from C_m by identifying an end vertex of the basic path in T_n at a vertex of C_m.

Definition 1.7. The Balloon of the Quadrilateral snake $Q_n(C_m)$ is the graph obtained from C_m by identifying an end vertex of the basic path in Q_n at a vertex of C_m.

Theorem 1.8. Any **Path** is Super Stolarsky-3 mean graph.

Theorem 1.9. Any Cycle is Super Stolarsky-3 mean graph.

Theorem 1.10. A Comb graph$(P_n \odot K_1)$ is Super Stolarsky-3 mean graph.

Theorem 1.11. The Ladder $L_n = P_2 \times P_n$ is Super Stolarsky-3 mean graph.

2. Main Results

Theorem 2.1. Any Triangular ladder graph Tl_n is Super Stolarsky-3 mean graph.

Proof: Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be two paths of length 'n'. Join u_iv_i, $1 \leq i \leq n$ and u_iv_{i+1}, $1 \leq i \leq n-1$. Then the resulting graph is Tl_n.

Define a function f : V(G)→{ 1,2,…., p+q} by $f(u_i) = 6i-5$, $1 \leq i \leq n$, $f(v_i) = 6i-3$, $1 \leq i \leq n$.

Then the edges are labeled with $f(u_iu_{i+1}) = 6i - 2$, $1 \leq i \leq n-1$, $f(v_iv_{i+1}) = 6i$, $1 \leq i \leq n-1$, $f(u_iv_{i+1}) = 6i - 1$, $1 \leq i \leq n-1$, $f(u_iv_i) = 6i - 4$, $1 \leq i \leq n-1$.

Thus the edge labels are distinct. Hence { f(V (Tl_n) ∪ f(E(Tl_n))} = {1, 2, …, p + q}.

Hence Triangular Ladder Tl_n is Super Stolarsky-3 Mean graph.

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Example 2.2. The Super Stolarsky-3 Mean labeling of Triangular Ladder Tl_6 is given below

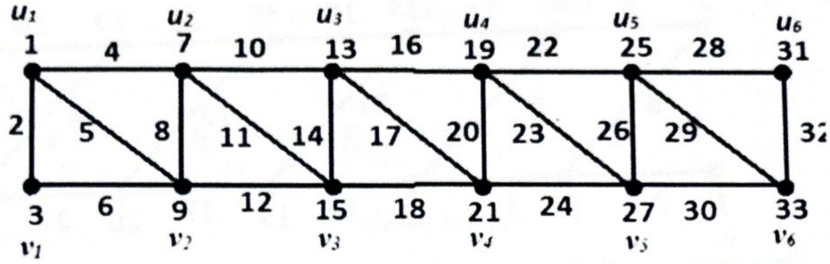

Figure 1

Theorem 2.3. Any Slanting ladder graph Sl_n is Super Stolarsky-3 mean graph.

Proof. Let G=SL_n be the slanting ladder graph with the vertices $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$.

Case(i) n is even

Define a function **f** : V(G)→{ 1,2,…., p+q} by $f(u_1) = 5$, $f(u_{2i-2}) = 10(i\text{-}1)\text{-}2$, $2 \le i \le \frac{n+2}{2}$, $f(u_{2i+1}) = 10i+3$, $1 \le i \le \frac{n-2}{2}$, $f(v_1) = 1$, $f(v_2) = 3$, $f(v_{2i+1}) = 10i$, $1 \le i \le \frac{n-2}{2}$, $f(v_{2i+2}) = 10i+5$, $1 \le i \le \frac{n-2}{2}$. Thus the induced edge labels are $f(u_iu_{i+1}) = 5i+1$, $1 \le i \le n-2$, $f(v_1v_2) = 2$, $f(v_iv_{i+1}) = 5(i\text{-}1)+2$, $2 \le i \le n-1$, $f(u_iv_{i+1}) = 5i\text{-}1$, $1 \le i \le n-1$. Then the edge labels are distinct.

Case (ii) n is odd

Define a function **f** : V(G)→{ 1,2,…., p+q} by $f(u_1) = 5$, $1 \le i \le n$, $f(u_{2i-2}) = 10(i\text{-}1)\text{-}2$, $2 \le i \le \frac{n+1}{2}$, $f(u_{2i+1}) = 10i+3$, $1 \le i \le \frac{n-1}{2}$, $f(v_1) = 1$, $f(v_2) = 3$, $f(v_{2i+1}) = 10i$, $1 \le i \le \frac{n-1}{2}$, $f(v_{2i+2}) = 10i+5$, $1 \le i \le \frac{n-3}{2}$. Then the edges are labeled as $f(u_iu_{i+1}) = 5i+1$, $1 \le i \le n-1$, $f(v_1v_2) = 2$, $f(v_iv_{i+1}) = 5(i\text{-}1)+2$, $2 \le i \le n-1$, $f(u_iv_{i+1}) = 5i\text{-}1$, $1 \le i \le n-1$.

Then the edge labels are distinct. Hence { f(V (Sl_n) ∪ f(E(Sl_n)} = {1, 2, …, p + q}.

From Case (i) and (ii), we conclude that Slanting Ladder graph SL_n is Super Stolarsky-3 Mean graph.

Example 2.4. The Super Stolarsky-3 Mean labeling of SL_6

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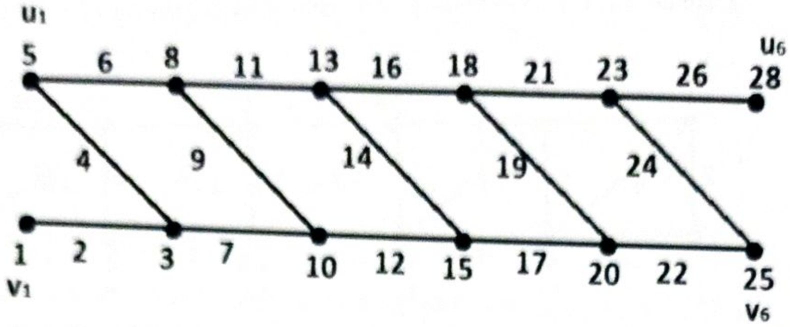

The Super Stolarsky-3 Mean labeling of SL_5

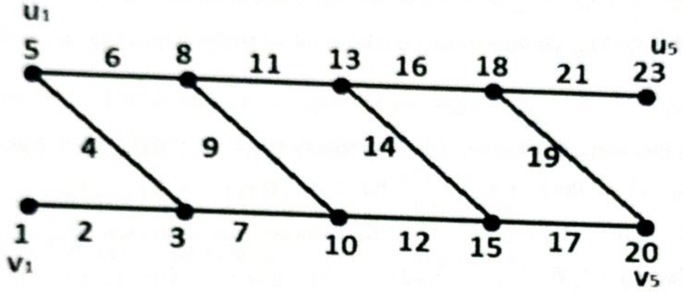

Figure 2: SL_6 and SL_5

Theorem 2.5. The Total graph $T(P_n)$ is Super Stolarsky-3 mean graph.

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n. *Let* $v_1, v_2, \ldots, v_{n-1}$ be its edges.

Let G= T (P_n). Here V (G) = V (P_n) ∪ E (P_n) and $E(G) = \begin{cases} u_i u_{i+1}, 1 \le i \le n-1. \\ u_i v_i, 1 \le i \le n-1. \\ u_i v_{i-1}, 2 \le i \le n. \\ v_i v_{i+1}, 1 \le i \le n-2. \end{cases}$

Define a function f : V(G) → {1,2,……,p+q} by $f(u_1) = 3$, $f(u_2) = 7$, $f(u_3) = 13$, $f(u_i) = 6(i-3)+12$, $4 \le i \le n$, $f(v_1) = 1$, $f(v_2) = 9$, , $f(v_3) = 15$, , $f(v_i) = 6(i-3)+15$, $4 \le i \le n-1$. Then the edges are labeled as $f(u_1u_2) = 5$, $f(u_iu_{i+1}) = 6(i-1)+4$, $2 \le i \le n-1$, $f(u_iv_i) = 6i-4$, $1 \le i \le n-1$, $f(u_2v_1) = 4$, $f(u_iv_{i-1}) = 6(i-2)+5$, $3 \le i \le n$, $f(v_1v_2) = 6$, $f(v_2v_3) = 12$, $f(v_3v_4) = 19$, $f(v_iv_{i+1}) = 6(i-3)+19$, $4 \le i \le n-2$. Then all the edge labels are distinct. Here { f(V (G) ∪ f(E(G)} = {1, 2, …, p + q}.

This gives T (P_n) is Super Stolarsky-3 Mean graph.

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Example 2.6. The Super Stolarsky-3 mean labeling of Total graph $T(P_5)$ is shown below.

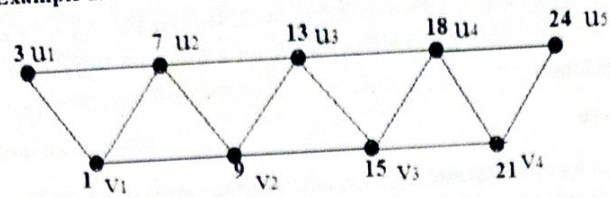

Figure 3: $T(P_5)$

Theorem 2.7. The Middle graph of the path $M(P_n)$ is Super Stolarsky-3 mean graph.

Proof. Let $u_1, u_2, \ldots, u_n$ & $v_1, v_2, \ldots, v_{n-1}$ be the vertices and edges of the Path P_n.

Let $G = M(P_n)$. By definition of middle graph $V(G) = V(P_n) \cup E(P_n)$ and whose edge set is

$$E(G) = \begin{cases} u_i v_i, & 1 \le i \le n-1 \\ u_i v_{i-1}, & 2 \le i \le n \\ v_i v_{i+1}, & 1 \le i \le n-2 \end{cases}$$

We define the function $f: V(G) \rightarrow \{1,2,3,\ldots,p+q\}$ by $f(u_1) = 1$, $f(u_i) = 5(i-1)$, $2 \le i \le n$, $f(v_i) = 5i-2$, $1 \le i \le n-1$. Then the distinct edge labels are $f(u_i v_i) = 5i - 3$, $1 \le i \le n-1$, $f(u_i v_{i-1}) = 5(i-1)-1$, $2 \le i \le n-1$, $f(v_i v_{i+1}) = 5i+1$, $1 \le i \le n-2$. Therefore $\{f(V(G)) \cup f(E(G))\} = \{1, 2, \ldots, p+q\}$. Hence Middle graph $M(P_n)$ is super stolarsky-3 Mean graph.

Example 2.8. The Super Stolarsky-3 Mean labeling of $M(P_6)$ is given below

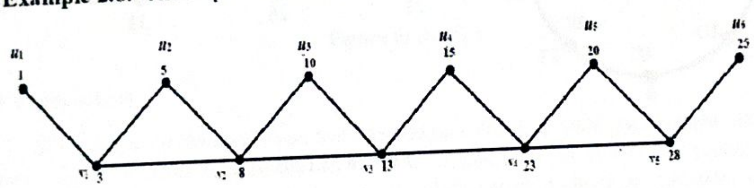

Figure 4: $M(P_6)$

Theorem 2.9. The Balloon of Triangular snake graph $T_n(C_m)$ is Super Stolarsky-3 mean graph.

Proof . Case (i) m is odd

Let $u_1, u_2, \ldots, u_m$, $v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_{n-1}$ be the vertices and u_1u_2, u_iu_{i+2}, i=2,4,6,....,n-3, $u_{n-1}u_n$, u_iu_{i+2}, i=1,3,5,....n-2 , v_iv_{i+1}, $1 \le i \le n-1$, v_iw_i, $1 \le i \le n-1$, w_iv_{i+1}, $1 \le i \le n-1$ be the edges of $T_n(C_m)$.

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Define a function $f: V(T_n(C_m)) \to \{1, 2, \ldots, p+q\}$ by $f(u_1) = 1$, $f(u_i) = 2i$, $i = 2, 4, 6, \ldots, m-1$, $f(u_3) = f(u_1) + 4$, $f(u_i) = f(u_{i-2}) + 4$, $i = 5, 7, 9, \ldots, m-2$, $f(u_n) = f(u_{n-1}) + 2$, $f(v_1) = f(u_m)$, $f(v_i) = f(v_{i-1}) + 5$, $2 \leq i \leq n$, $f(w_1) = f(v_1) + 2$, $f(w_i) = f(w_{i-1}) + 5$, $2 \leq i \leq n-1$. Then we get distinct edge labels.

Case (ii) m is even

Let $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_{n-1}$, be the vertices and u_1u_2, u_iu_{i+2}, $i = 2, 4, 6, \ldots, n-2$, $u_{n-1}u_n$, $u_{n-3}u_{n-1}$ u_iu_{i+1}, $i = 1, 3, 5, \ldots, n-5$, v_iv_{i+1}, $1 \leq i \leq n-1$, v_iw_i, $1 \leq i \leq n-1$, w_iv_{i+1}, $1 \leq i \leq n-1$ be the edges of $T_n(C_m)$.

Define a function $f: V(T_n(C_m)) \to \{1, 2, \ldots, p+q\}$ by $f(u_1) = 1$, $f(u_i) = 2i$, $i = 2, 4, 6, \ldots n$, $f(u_3) = f(u_1) + 4$, $f(u_i) = f(u_{i-2}) + 4$, $i = 5, 7, 9, \ldots, n-1$, $f(v_1) = f(u_m)$, $f(v_i) = f(v_{i-1}) + 5$, $2 \leq i \leq n$, $f(w_1) = f(v_1) + 2$, $f(w_i) = f(w_{i-1}) + 5$, $2 \leq i \leq n-1$. Then the edge labels are distinct.

By the above two cases $\{f(V(T_n(C_m)) \cup f(e)\} = \{1, 2, \ldots, p+q\}$. Hence The Balloon of Triangular snake graph $T_n(C_m)$ is Super Stolarsky-3 mean graph.

Example 2.10. Super Stolarsky-3 mean labeling of $T_4(C_8)$ is given below

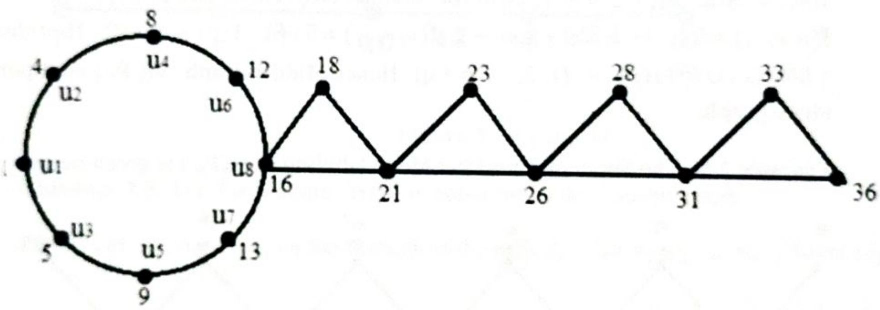

Figure 5: $T_4(C_8)$

Theorem 2.11. The Balloon of Quadrilateral snake graph $Q_n(C_m)$ is Super Stolarsky-3 mean graph.

Proof Case (i) m is odd

Let $u_1, u_2, \ldots, u_m$, $v_1, v_2, \ldots, v_n$ and $x_1, x_2, \ldots, x_{n-1}$, $w_1, w_2, \ldots, w_{n-1}$ be the vertices and u_1u_2, u_iu_{i+2}, $i = 2, 4, 6, \ldots, n-3$, $u_{n-1}u_n$, u_iu_{i+2}, $i = 1, 3, 5, \ldots, n-2$, v_iw_i, $1 \leq i \leq n-1$, v_iv_{i+1}, $1 \leq i \leq n-1$, w_ix_i, $1 \leq i \leq n-1$, x_iv_i, $1 \leq i \leq n-1$) be the edges of $Q_n(C_m)$.

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Define a function $f : V(Q_n (C_m)) \rightarrow \{1,2,\ldots, p+q\}$ by $f(u_1) = 1$, $f(u_i) = 2i$, $i=2,4,6, \ldots, n-1$, $f(u_3) = f(u_1) + 4$, $f(u_i) = f(u_{i-2}) + 4$, $i=5,7,9,\ldots,n-2$, $f(u_n) = f(u_{n-1}) + 2$, $f(v_1) = f(u_m)$, $f(v_i) = f(v_{i-1}) + 7$, $2 \leq i \leq n-1$, $f(w_1) = f(v_1) + 2$, $f(w_i) = f(w_{i-1}) + 7$, $2 \leq i \leq n-1$.

$f(x_1) = f(w_1) + 3$, $f(x_i) = f(x_{i-1}) + 7$, $2 \leq i \leq n-1$.

Case (ii) m is even

Let $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_{n-1}$, be the vertices and $u_1 u_2$, $u_i u_{i+2}$, $i=2,4,6,\ldots,n-2$, $u_{n-1} u_n$, $u_{n-3} u_{n-1}$ $u_i u_{i+1}$, $i=1,3,5,\ldots,n-5$, $v_i v_{i+1}$, $1 \leq i \leq n-1$, $v_i w_i$, $1 \leq i \leq n-1$, $w_i v_{i+1}$, $1 \leq i \leq n-1$ be the edges of $Q_n (C_m)$.

Define a function $f : V(Q_n (C_m)) \rightarrow \{1,2,\ldots, p+q\}$ by $f(u_1) = 1$, $f(u_i) = 2i$, $i=2,4,6,\ldots,n$, $f(u_3) = f(u_1) + 4$, $f(u_i) = f(u_{i-2}) + 4$, $i=5,7,9,\ldots,n-1$, $(v_1) = f(u_m)$, $f(v_i) = f(v_{i-1}) + 7$, $2 \leq i \leq n-1$, $f(w_1) = f(v_1) + 2$, $f(w_i) = f(w_{i-1}) + 7$, $2 \leq i \leq n-1$, $f(x_1) = f(w_1) + 3$, $f(x_i) = f(x_{i-1}) + 7$, $2 \leq i \leq n-1$.

From the above two cases $\{f (Q_n (C_m) \cup f(e)\} = \{1, 2, \ldots, p + q\}$. Hence the Balloon of Quadrilateral snake graph $Q_n (C_m)$ is Super Stolarsky-3 mean graph.

Example 2.12. The Labeling pattern of Stolarsky-3 mean labeling of $Q_3 (C_7)$ is given below

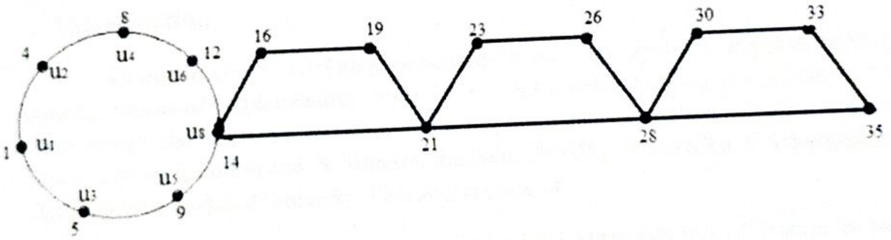

Figure 6: $Q_3 (C_7)$

3. Conclusion

In this paper we discussed Super Stolarsky-3 Mean Labeling of some special graphs. All graphs are not Super Stolarsky-3 mean labeling of graphs. In this paper, we proved that Triangular ladder graph, Slanting ladder graph, Total graph, Middle graph, Balloon of Triangular snake graph and Balloon of Quadrilateral snake graph are Super Stolarsky-3 Mean graphs. It is possible to investigate similar results for several other graphs.

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